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Abelian surfaces of type $(1, 4)$.

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Let A denote an abelian surface over the complex numbers and L an ample line bundle on A . We may think of A as the quotient of $\mathbf{C}^2$ by a lattice Λ . The first Chern class of L may then be viewed as an integer-valued alternating form on Λ . A theorem of Kronecker associates to this alternating form a pair of positive integers (d_1, d_2) such that d_1 divides d_2 . The Riemann-Roch theorem then says that L induces a rational map $\varphi: A \rightarrow \mathbf{P}_{d_1 d_2 - 1}$. This paper investigates the structure of this map in the case $d_1 = 1$, $d_2 = 4$. The main result states that the image of φ is either a birational map of A onto a singular octic in $\mathbf{P}_3$ or a double covering of a singular quartic which is birational to an elliptic scroll. Another result of this paper is that the moduli space for the pairs (A, L) as above is the quotient of a rational 3-fold by the action of the symmetric group of order 24. A lot of beautiful geometry is developed along the way to these results.

Reviewed by [David Ortland](#)

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